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Mathematics News Letter

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Published under auspices of the Louisiana-Mississippi Section of the Mathematical Association of America and the Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics.

To mathematics in general, to the following causes in particular is this journal dedicated: (1) the common problems of grade, high school and college mathematics teaching, (2) the disciplines of mathematics, (3) the promotion of M. A. of A. and N. C. of T. of M. projects.

VOL. 4 BATON ROUGE, LA., OCTOBER, 1929 NO. 2

URGENT NEEDS OF THE NEWS LETTER

The Mathematics News Letter is now honored with paid-up subscriptions in 21 States of the Union. The librarians of a number of colleges and high schools have subscribed. These facts should be, and doubtless are, distinctly encouraging to both Section and Council officials in Louisiana-Mississippi territory. Especially is the News Letter editorial management deeply appreciative of the sympathetic attitude assumed by mathematical workers representing nearly every part of the country.

But our needs are still extremely urgent. A far more rapid income of subscription dollars is necessary if we who are responsible for the publication and distribution of the journal are to be able to meet the monthly costs of printing and mailing it out from Baton Rouge. Chairman Hardin, of Shreveport, Secretary Dale, of Cleveland, are making heroic efforts to carry out the scheme adopted at Lafayette for developing and maintaining a bi-State-wide financial and moral support of our monthly. But, unless there is a far more general "holding up of their hands" on the part of the great body of district leaders who have been appointed in the two States, these costs cannot be met by the "subscription" plan.

Another crying need is a greatly enlarged staff of contributors to the News Letter. It has reached our editorial ears that we are being criticized because too much of the contributed material is being furnished by Louisiana State University mathematicians, and too little by high school and college people in other sections. Steady readers of the News Letter will see at once the groundlessness of such criticism. Again and again have we invited—almost pleaded with—our college and high school clientele to send us their articles for use in our pages. Most of the time our pleadings have been ignored or passed over, notwithstanding they have been made both editorially and by personal or circular correspondence. We repeat. *A crying need is a greatly enlarged staff of contributors to the News Letter.*

—*S. T. S.*

NICHOLSON'S TRIGONOMETRIC CIRCLE

How many mathematicians have heard of it? According to our own observation made over a period of twenty years we may answer: Very few indeed. Yet it is of such practical value as a memory device for the student of plane trigonometry that we have repeatedly expressed our surprise that writers of trigonometries have in no case that we recall requested the author of it (Colonel J. W. Nicholson) to allow its publication by them in their own texts. Colonel Nicholson had embodied it in his own text—an excellent publication and one widely used many years ago.

Suppose a right triangle with legs denoted by a and b, hypotenuse by c. If the six trigonometric functions are referred to the angle opposite a, say A, and if a circular arrangement is made of them taking them in such an order that each pair of successive functions have either a common numerator or a common denominator the following sequence results: a/c, b/c, b/a, c/a, c/b, a/b, a/c, etc. Translating it into trigonometric language it becomes, omitting for brevity's sake the angle symbol "A": Sine, cosine, cotangent, cosecant, secant, tangent, sine, etc. Since the sequence of ratios has the property that each successive pair has either a common numerator or a common denominator, it is easily seen that if any one of them should be divided into the ratio adjacent to it the quotient must be the ratio ad-

jacent to that one and two places removed from the dividing ratio. This will be true without regard to whether the adjacent ratio is taken on the right or on the left. Put otherwise, the product of any two alternate ratios is equal to the ratio between them. Again it is seen also that the product of two ratios so placed in the sequence that two other ratios are between them is equal to 1. It follows, of course, that the function sequence, namely, sine, cosine, cotangent, cosecant, secant, tangent, sine, etc., has the same properties. Thus, the student has only to commit the function sequence to memory and apply the two described rules (properties) in order quickly to determine what two functions multiplied together will yield a third, or unity, and what two functions divided will yield a third one. Some may think it just as well to write the function sequence in circular order applying the two mentioned rules while looking at the written arrangement.

Not less important than the mnemonic value of the trigonometric circle, is that the scheme is scientifically exhaustive. The precise number of all possible products and quotients which are expressible as a single function is made evident to the student in a very simple manner. —*S. T. S.*

SUGGESTIONS FOR IMPROVING THE TEACHING OF SOLID GEOMETRY

By JOHN R. SHOPTAUGH
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This paper attempts to give suggestions for improving the teaching of solid geometry. The suggestions are given for the purpose of a more thorough realization of the aim as indicated in the following statement: "The aim of the work in solid geometry should be to exercise further the spatial imagination of the student and to give him both a knowledge of the fundamental spatial relationships and the power to work with them. It is felt that the work in plane geometry gives enough training in logical demonstration to warrant a shifting of emphasis in the work on solid geometry away from this aspect of the subject and in the direction of developing greater facility in visualizing

spatial relations and figures, in representing such figures on paper, and in solving problems in mensuration."[1]

Pupils often experience difficulty in visualizing three dimensional figures from drawings. Furthermore, correct spatial concepts are built by the use of both the eye and hand. By building spatial concepts is meant analytical spatial concepts and ability to represent the whole out of its analyzed parts. Hence, it is desirable, especially in the early part of the work, to make three dimensional constructions as an aid in the study of the theorems. Furthermore, "greater facility . . . . in representing such figures on paper" is obtained when the pupil makes drawings of his constructions. By comparing his drawings with his constructions and with the drawings in the textbook, he readily develops the ability to represent three dimensional figures on paper.

I have selected, for purposes of illustration, a few of the theorems from the first book in solid geometry. My reason for doing this is that the development of spatial imagination and in some degree the power to use these fundamental spatial relationships should come early in the course and should be the product largely of the study of these preliminary theorems. The development of spatial imagination should be stimulated, of course, throughout the remaining books of the course. This should occur not only through a study of geometrical relationships as may be indicated on a flat surface, but by actual construction out of cardboard or other materials all the geometrical solids studied. As a further aid in realizing this objective, frequent references should be made to the geometrical figures found in nature and in the works of man.

It must be understood that the material included here is only suggestive. Classes and classroom conditions differ. It is evident that for some classes different material may be used more advantageously. In some classes it may be found desirable to continue the type of construction I have indicated so as to apply to all preliminary theorems of the first book of the text. Other classes or individuals in a class may need to do considerably less. When a pupil or class of pupils has developed suf-

[1]The Reorganization of Mathematics in the Secondary Schools, p. 31, Bulletin No. 32, 1921, Dep't of Interior, Bureau of Education, Washington, D. C.

ficient spatial imagination so that he or they can see and think three dimensions from a drawing on a flat surface, these 'crutches' of construction may be dropped. The teacher of necessity must be the judge in this matter. The constructions should be left in so far as it is practical to the ingenuity of members of the class. The construction itself presents a valuable problem. It calls forth intrinsic interest and enthusiastic cooperation. Pupils feel that it is a natural method of approach to the solution of the problem presented by the theorem. Pupils who are considered poor in mathematics are often superior in solving problems of construction. The feeling of success and the warmth of interest accompanying it readily diffuse over into other and more abstract phases of the work. The detailed suggestions given here are to be used by the teacher as an aid, when needed, to stimulate pupils' initiative and thinking.

Proposition I. If a line is perpendicular to each of two intersecting lines at their point of intersection, it is perpendicular to the plane of the two lines.

Give pupils thread and stick with holder.

Problem: To make construction needed for proof.

Suggestion 1. Stand stick on table directly over table leg.

Suggestion 2. Run thread from bottom of holder to top of chair, making approximately a right angle formed by the thread and the stick. Label stick at top A; at bottom B; thread at chair, D.

Suggestion 3. Run a thread from the bottom of the stick to another chair forming approximately a right angle with the stick. Label the point at which this thread is attached to chair, C.

Question 1. What does each of the threads represent?

Question 2. What does the stick represent?

Suggestion 4. From the bottom of the stick run a third thread within the angle formed by the other two threads and in the same plane and tie to a chair at a greater distance from the bottom of the stick than either of the other two chairs used. Label the point at which this thread is attached to chair, X. To get the third thread in the same plane as the other two, have pupils sight over the first two threads and raise or lower the third thread at point X until it appears to be in the same plane.

Question 3. What does this third thread BX represent?

Question 4. What shall we need to prove if we prove the theorem?

Suggestion 5. Run a thread from D to C. It should touch the thread BX. Label the point of intersection of BX and DC, G.

Question 5. Why should DC just touch BX?

Suggestion 6. Measure from the bottom of the stick downward on the table leg a distance approximately equal to AB. Label this point F. Run threads from this point to D, G, and C. Also run threads from A to D, G, and C.

Question 6. What angle is to be proved a right angle, if we prove the theorem?

Question 7. How many triangles have we constructed?

Question 8. What lines are equal?

Question 9. What angles are equal?

Question 10. What triangles are congruent?

Proof of theorem: See text.

Theorem: Through a given point in a plane there can be drawn one line perpendicular to a plane, and only one.

Suggestion 1. Let the floor represent a plane and let a stick represent a line. Make a mark on the floor with crayon representing a point, A. Have pupil hold stick on point A, so that the stick will be approximately perpendicular to the floor. Have pupil then put another stick on point A, or approximately so, and attempt to hold it also so that it will be perpendicular to the floor.

Question 1. What happens?

Question 2. What do you conclude from this?

Give pupils a piece of cardboard of dimensions about 12 inches by 18 inches. Ask them to consider the floor a plane.

Problem: To make construction needed for proof.

Suggestion 2. Draw with crayon any straight line, CD, through the point A.

Question 3. How many sticks could be placed at point A and be perpendicular to CD?

Question 4. What would all of these determine?

Question 5. How would you represent this?

Suggestion 3. Stand the cardboard on edge so that it will pass through A and be perpendicular to CD. Stack books at each end of cardboard to hold it in position. This cardboard represents

a part of the plane above the floor which is perpendicular to DC.

Question 4. Will all lines drawn in the plane of the cardboard and passing through A be perpendicular to CD?

Question 5. Can a line be drawn perpendicular to CD at point A and not lie in the plane represented by the cardboard?

Suggestion 4. Call the line formed by the intersection of the cardboard with the floor RS.

Question 6. Is RS perpendicular to DC at point A?

Suggestion 5. Draw a line, AK, in the cardboard perpendicular to RS and passing through A.

Question 7. Will AK be perpendicular to DC also?

Question 8. Are DC and RS in the same plane?

Proof of theorem: See text.

Theorem: Through a given external point there can be drawn one line perpendicular to a given plane, and only one.

Suggestion 1. Consider the floor a plane. Let the corner of a table represent the external point, A. Use strings to represent lines.

Question 1. In terms of table corner, floor, and string, what are you to prove?

Give pupils string, pins, crayon, and a meter stick or yard stick for a straight edge.

Problem: To make construction needed for proof.

Suggestion 2. On the floor at a convenient angle and convenient distance from the corner of the table (point A) draw with crayon any straight line BX. Imagine a plane standing on BX and leaning against point A. Attach a string by means of a thumb tack to A. Do not press the tack too far in, as you will need to attach other strings to it. Tack the other end of the string to line BX at point D that it will be approximately perpendicular to BX.

Suggestion 3. On the floor at D draw a line DY perpendicular to BX. Now AD and DY determine a plane.

Question 2. Will any line drawn from A to DY be in that plane?

Question 3. Can a perpendicular be drawn from A to DY?

Suggestion 4. From A to DY run a string approximately perpendicular to DY. Call the point where the string is attached to DY, L. Then AL is perpendicular to DL.

Question 4. Do we know it to be perpendicular to the floor?

Suggestion 5. From D on DX measure off a distance DC equal to AL. Run a string from A to C and draw a line from L to C.

Question 5. How many right triangles have we constructed?

Question 6. Does ALC appear to be a right triangle?

Suggestion 6. AL was constructed perpendicular to LD. If triangle ALC is a right triangle, then AL will be perpendicular also to LC and therefore perpendicular to the floor.

Suggestion 7. Name lines in triangle ALC and triangle ACD that are equal.

Question 7. If LC equals AD, then triangle ALC and triangle ADC would be congruent. Would AL of necessity then be perpendicular to LC?

Suggestion 8. LC equals AD if triangle LCD and triangle ALD are congruent.

Question 8. Is triangle LCD congruent to triangle ALD? Name the parts of one that have corresponding equal parts in the other.

Proof: See text.

Theorem: Oblique lines drawn from a point to a plane, meeting the plane at equal distances from the foot of the perpendicular, are equal; and of two oblique lines meeting the plane at unequal distances from the foot of the perpendicular, the one corresponding to the greater distance is the greater.

Give pupils thread, stick, with holder, and crayon. Consider the floor a plane.

Problem: To make construction needed for proof.

Sugestion 1. Stand the stick upright in holder and place on floor. Label top of stick A and bottom of stick B. Then AB represents a line perpendicular to the plane of the floor.

Suggestion 2. Measure off any convenient length of string and attach one end loosely at the base of the stick (at B). With this length as a radius and B as a center, draw on the floor a circle.

Suggestion 3. Take any two convenient points on this circle. Label one of the points D and the other one C. Draw lines from D to B and from C to B. Run threads from C to A and from D to A.

Question 1. What must we prove if we prove the first part of the theorem?

Suggestion 4. Select a convenient point on the floor outside of the circle. Label this point H. Run thread from H to A and draw line from H to B, where HB intersects the circumference of circle label F. Run thread from F to A.

Question 2. What must we prove if we prove second part of this theorem?

Suggestion 5. How does AH compare in length with AF? With AC? With AD?

Proof: See text.

Two lines perpendicular to the same plane are parallel.

Give pupils thread, crayon, two sticks with holders, and thumb tacks. Consider the floor a plane and the sticks standing upright in holders on the floor as lines perpendicular to the floor.

Problem: To make construction needed for proof.

Suggestion 1. Attach labels to the top and bottom of each stick, labeling one stick AB and the other CD. Draw on the floor the line BD.

Question 1. AB and CD are each perpendicular to the floor. Are each perpendicular to BD?

Question 2. If we knew AB and CD to be in the same plane, would they necessarily then be parallel?

Suggestion 2. All we need to do then is to make such construction as will aid us in proving AB and CD to be in the same plane.

Suggestion 3. Through D draw on the floor XY perpendicular to BD. From the foot of the perpendicular CD cut off equal distances on each side of it along the line XY, and label these parts DF and DH.

Suggestion 4. Run thread from F to A and draw line from F to B. Run thread from H to A and draw line from H to B. Run thread from D to A and draw line from D to B.

Question 3. What lines are equal?

Question 4. What lines are perpendicular?

Question 5. If BD, AD, and CD are each perpendicular to HF at point D, would AB and CD then of necessity be in the same plane?

Proof: See text.

Theorem: If two lines are parallel, every plane containing one of the lines, and only one is parallel to the other.

Suggestion 1. Find two lines in the room that are parallel.

Suggestion 2. Find a line in a plane and a second line not lying in this plane but parallel to the first line.

Question 1. Is the plane (Suggestion 2) parallel to the second line?

Consider the floor a plane.

Problem: To make construction needed for proof.

Suggestion 3. Draw a line on the floor or if there is a crack in the floor, consider it a line in the given plane. Run a thread from the top of one chair to the top of another so that the thread will be approximately parallel to the line on the floor.

Suggestion 4. Consider a plane passed through the thread and the line in the floor. As an aid to visualizing this plane, run threads from each point of attachment on chairs approximately perpendicular to the line in the floor.

Proof: See text.

Theorem: If two planes are perpendicular to the same straight line, they are parallel.

Problem: To make construction needed for proof.

Suggestion 1. Do you see any two planes in the room that are perpendicular to the same line? Let the floor and the table top each represent planes. Let the table leg represent a line to which each of the two planes is perpendicular.

Question 1. If the floor and table top are not parallel, and if each were extended out without limit, would they meet? What kind of geometrical figure would be formed, if any?

Question 2. Suppose you selected any point in the line formed by the intersection of the floor and the table top. Could a perpendicular be drawn from this point to the line represented by the table leg and lie in the same plane represented by the table top? Could a perpendicular be drawn also from this point to the line represented by the table leg and lie in the plane represented by the floor?

Suggestion 2. You would then have two perpendiculars from a point to a line, which is impossible. Where is the difficulty?

Proof: See Text.

Theorem: The intersection of two parallel planes by a third plane are parallel lines.

Question 1. Are there two planes in this room that are

parallel and that intersect a third plane? Do the straight lines formed appear to be parallel?

Question 2. Suppose the two straight lines formed are not parallel. What would you say about the two planes you assumed to be parallel?

Suggestion 1. Have pupils make model from pasteboard for the construction needed for proof.

Proof: See text.

Theorem: If two angles not in the same plane have their sides respectively parallel and lying on the same side of the straight line joining their vertices, the angles are equal, and their planes are parallel.

Suggestion 1. Consider the floor one of the planes. Use stick to represent the line joining the vertices of the two angles.

Problem: Make construction needed for proof.

Suggestion 2. On the floor draw any convenient acute angle, XAY. At A stand stick upright.

Suggestion 3. Label upper end of stick B. From B run thread approximately parallel to AX and attach to a chair. Call this point on chair C. Then BC represents a line parallel to AX.

Suggestion 4. From B run a thread approximately parallel to AY and attach to a chair. Label this point of attachment D. Then BD represents a line parallel to AY.

Question 1. What now are we to prove?

Suggestion 4. From along AX take a definite length AM. From B along BC take a length BN equal to length AM. Run a thread from N to M.

Question 2. What do you know about the figure ABNM?

Suggestion 5. From A along AY take a definite length AP. From BX along BD take a length BQ equal to length AP. Run a thread from Q to P.

Question 3. What kind of figure is ABQP?

Suggestion 6. Draw a line from P to M and run a thread from Q to N.

Question 4. What do you know about the figure PQNM?

Proof: See text.

Theorem: If two lines are cut by three parallel planes, their corresponding segments are proportional. Use threads for lines.

Suggestion 1. Place chair by the side of table. Consider

table top, seat of chair, and the floor, as three parallel planes.

Problem: Make construction needed for proof.

Suggestion 2. Attach thread at edge of top of table, labeled point A, and run to floor, labeled point B, so as to just touch edge of chair seat, labeled point E. ABC then represents a straight line cut by the three parallel planes forming line segments AE and ED.

Suggestion 3. Select some point, C, near the center of the table top. Attach thread on under side of table at this point and run to near center of chair seat. Label this latter point F. Attach thread on under side of chair seat at point F and run thread in continuation of the straight line, CF, until it touches the floor. Label point where thread touches floor, D. Then CFD represents a straight line cut by the three parallel planes forming line segments CF and FD.

Suggestion 4. Draw on top of table with crayon, line AC. Run thread from A to D, piercing chair at G. That is, select a point G on the seat of the chair in same straight line with A and D, and run thread from A to G. Then attach thread on the under side of the chair at point G and run thread to point D.

Suggestion 5. Draw GF with crayon on top of chair seat.

Question 1. In the triangle ABD, is EG parallel to BD? Why?

Question 2. In the triangle ADC, is GF parallel to AC? Why?

Proof: See text.

MULTUM IN PARVO

By GEO. F. WILDER
New York, N. Y.

Truly wonderful is the power of the symbolism of mathematics to represent in concise, visible form numerical complexity and immensity, and this even to a degree beyond the power of the mind to conceive. For example, suppose one were to attempt to represent the greatest possible number by the use of the integers 1, 1, 1, and 0, 0, 0,—these to be combined in any way not violating the accepted usages of mathematics. The solution

would be found by writing $10^{10^{10}}$. The enormity of numerical magnitude represented in this simple form is difficult to comprehend. Since 10^n equals 1 followed by n zeros, 10^{10} equals 10,000,000,000 or 10 billions. Accordingly, $10^{10^{10}}$ equals 1 followed by **10 billion zeros,** a number which staggers the imagination. If we were to attempt to write out the number as on a blackboard, putting, let us say, one figure to the inch, we should require a blackboard nearly 160,000 miles in length; if we were to write 5 figures per second, and kept continuously at the task, we might hope to complete it in about 63 years. To aid our imagination in an attempt to conceive of the magnitude represented, let us consider the sands of the sea, which are usually popularly represented as "countless", and let us not limit ourselves to the seashore, but be liberal and consider the whole earth to be composed of very fine sand, each grain a cube $^1/_{100}$ of an inch on a side packed solidly together, and the earth a cube 8000 miles on a side. The number of sand grains under these conditions would be $(8000 \times 5280 \times 12 \times 100)^3$ or approximately 13 followed by only 31 zeros. If we write this number on the same scale as above, our blackboard need be but 33 inches long as compared with 160,000 miles for $10^{10^{10}}$. The number of sand grains sinks to insignificance on the yardstick we are using. Let us seek a larger cube than the earth affords. Astronomers nowadays in discussing island universes do not hesitate at distances of 1,000,000 light years. If our cube were of this dimension, and we take for the velocity of light 200,000 miles per second (instead of Prof. Michelson's nearly exact figure of 186508 miles), the number of sand grains would then be $(1{,}000{,}000 \times 365 \times 24 \times 60 \times 60 \times 200{,}000 \times 5280 \times 12 \times 100)^3$, or approximately 64 followed by 75 zeros. We now need a blackboard 77 inches long, but our number still sinks to insignificance. Let us make one last attempt to a physical approximation of $10^{10^{10}}$. We will combine the macrocosm of the astronomer with the microcosm of the physicist. If the million year light cube were filled with matter composed of atomic cubes one trillion to the inch, we should have for the number of atoms.

$(1{,}000{,}000 \times 365 \times 24 \times 60 \times 60 \times 200{,}000 \times 5280 \times 12 \times 1{,}000{,}000{,}000{,}000)^3$ or approximately 64 followed by 105 zeros. We would

now require a blackboard 107 inches in length, and even now our number is as nothing compared with $10^{10^{10}}$. The physical universe seems not to yield an illustration. Only in imagination can we compass it. The mechanism of mathematics has surely evolved a long way from primitive man's method of utilizing his fingers and toes for the first twenty numbers, and being content with "many" for all greater ones.

A SOLUTION TO THE THEOREM OF APOLLONIUS

CHESTER R. HILLARD
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The third century before Christ produced three of the greatest mathematicians of antiquity, namely Euclid, Archimedes and Apollonius, all of whom were very much interested in the field of geometry. The third great mathematician of the century was Apollonius of Perga, who gave to the world a systematic treatise on conic sections, thus extending immensely the existing knowledge of these curves. He was the author of a problem "Given two coplanar straight-lines A*a* and B*b*, drawn through fixed points A and B; to draw a line O*ab* from a given point O outside them cutting them in *a* and *b*, so that A*a* shall be to B*b* in a given ratio." According to Halley's translation from an Arabic copy, Apollonius reduced the problem to seventy-seven separate cases, and gave appropriate solutions for all, with the aid of conics. Apollonius later wrote a treatise "De Sectione Spatii" on the identical problem only under the condition that the rectangle A*a* · B*b* be given.

The writer came across this proof to the above theorem and presents it for the interest which it may lend to those interested in this field of mathematics.

Proof:

1. Angle DKG=angle JKM (All vertical angles are equal.)
2. Angle PLE=angle JKM (Supplements of same angle are equal.)
3. Angle DKG=angle JKM=angle PLE.
4. Angle NPG=angle NDG (Inscribed angles measured by half of same intercepted arc are equal.)

5. Angle NPG = angle PEL (Alternate-interior angles.)

6. Angle NPG = angle NDG = angle PEL.

7. Triangle DKG $\sim$ Triangle PLE (Two corresponding angles in each triangle are equal respectively.)

8. PL : KG = LE : DK (Corresponding sides of similar triangles are proportional.)

9. PL·DK = KG·LE (Product of means equals product of extremes.)

10. KG·LE = Rectangle R (Area of rectangle equals product of base and altitude.) *Q. E. D.*

Hypothesis: The two points K and L on the two fixed straight lines AB and AC; the external point P and any rectangle R.

Required: To draw a straight-line PEG intersecting AB and AC in points G and E such that KG·LE equals the rectangle R.

Construction: 1. Draw PL and let it intersect AB at the point J.

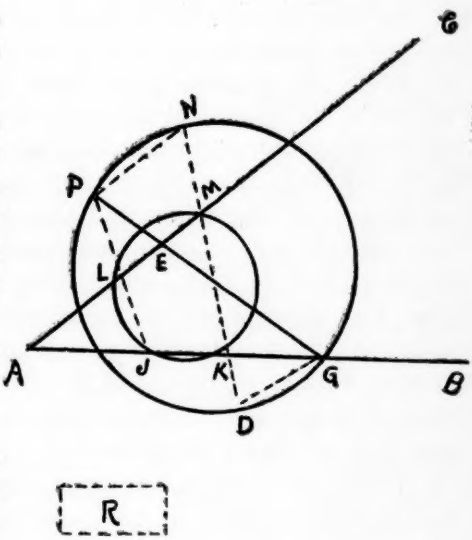
2. Through points J, K, and L describe a circle, intersecting AC at point M.

3. Through point P draw PN parallel to AC, meeting KM produced at the point N.

4. Take KD on MK so that KD·PL equals the rectangle R.

5. Through the points D, P, and N describe a circle intersecting AB at point G.

6. Draw DG and PEG.



Then, PEG is the required straight line cutting AB and AC at the points G and E so that KG·LE equals the rectangle R.

Q. E. F.

If every high school of Louisiana-Mississippi should subscribe to the News Letter the cost of a year's printing would be netted.

BEGINNERS' ALGEBRA—WHAT AND HOW?

By DORA M. FORNO
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The success of any undertaking depends upon two vitally significant facts:—first, a well-organized plan of what you expect to accomplish, with the necessary materials and tools needed; and, second, cooperation between all concerned to bring about desired results.

Pupils come up from the grades knowing little about the content of highi school subjects or their significance. Their background is limited and most frequently they are prejudiced in favor of or against a subject. Certain subjects would be tabooed if students had their say in the matter. Algebra is one of these subjects and has been classed as "hard" because of the number of failures among beginners. These failures in algebra can be assigned to one or more of three causes,—psychological, physical or pedagogical.

From the psychological point of view the student may be lacking in mathematical abilities—a deficiency which may or may not be corrected, according to whether they are inherent or due to lack of training. But the student may have approached algebra from a wrong angle and this is an important point in the teaching of the subject and must be considered. From the physical point of view, apart from any personal physical handicaps, the atmosphere of the classroom, including the physical situation and the teacher's attitude, has much to do with the advancement of a student in his progress in learning. An active spirit of cooperation betwen teacher and pupil is absolutely necessary to a student's progress.

From the pedagogical point of view, even with the advance that has been made in the teaching of mathematics, educators are confident that still more can be done to improve the pedagogy of the subject in order that greater interest may be aroused in the students and better results attained.

An "inspirational preview" or "a birds eye view" of the subject is one of the most valuable approaches to the study of algebra for awakening interest and putting before the class reasons for studying algebra and some of the things that are ex-

pected during the course. The important thing is to give the students an inviting introduction to it so that they will anticipate pleasure in the study of the subject. Then, it is the privilege of the instructor to see that this interest is kept up.

By means of formulas that students have become familiar with in arithmetic, algebra can be presented as a generalized arithmetic. It is more powerful than arithmetic, as it furnishes easier and more reliable ways of solving problems. It can be shown that algebraic equations have definite solutions and beginners can be shown that they can check their work very readily and have a positive assurance that their work is correct. It can be shown that the language of algebra makes reasoning easier, —that algebraic notation is a method of expressing numbers by letters and figures and that the number relations expressing opposite directions, kind or quality, can be indicated by positive and negative numbers, while in arithmetic size is the only relation that can be expressed. The important objective that must be made apparent is the use of algebra as a powerful tool for problem solving. The fact that algebra is a source of mental training and logical reasoning should be kept in the background.

The early work in algebra should not be done hurriedly The power to interpret the meaning of algebraic expressions and the power to express relations in algebraic language should be most carefully developed. Concrete presentations by means of effective devices make the work real. Many examples may be given to illustrate the nature of positive and negative numbers. The thermometer may be used successfully here. Indicate the reading above zero, positive and below, negative and then calculate the temperate according to changes or calculate changes from successive readings. Concrete situations may also be gotten from pupils' activities and interests. A boy who was having some difficulty with his algebra after having several days work on positive and negative numbers was made to see the significance of negative numbers by the use of the familiar expression "so many in the hole" and, thereafter, was able to compute readily in terms of signed numbers, for now they had some real meaning to him.

Errors in algebra are often due to wrong experiences or to lack of experiences from the first. The old adage that an ounce of prevention is worth a pound of cure is applicable to all learning. Much remedial work in algebra would be un-

necessary if the proper precautions were taken to lay the foundation of the subject on actual experience, so that students may be able to see the reason for certain procedures rather than rely on mere memorization of certain rules and formulas. After students grasp underlying principles, they are able to formulate rules for themselves. A background of reasoning serves as a bulwark at all times against future errors.

The "general lack of understanding of mathematical method" referred to by Dr. Webber in his article "Some Common Errors in Algebra" is due partly to lack of understanding of fundamental principles in the first learning and, also, to lack of proper drill provisions to fix those principles during the learning period. The drills are frequently weak to the extent that they give over-practice on simple phases of the work and little practice on those phases that give greater difficulty. It is of paramount importance that after new processes are taught and drilled upon in the first learning, systematic drill be pursued to maintain that skill which has been built up. I believe that much of the loss of skill in the simplest operations is due to the fact that not enough opportunities have been provided for using them.

The second aspect of elementary algebra to which I wish to refer is the use of the equation as the mode of expressing mathematical relations. It is the handy tool for solving equations. It has been called the "backbone of algebra." To handle this tool skillfully, it is necessary to learn certain fundamental principles governing the formation of equations and changes made in them. The power to interpret the meaning of problems varies so greatly in individuals that a large part of the work in elementary algebra consists in trying to gain improvement in reading comprehension. The next step is to gain power to express the data of the problem in algebraic language. This is a most difficult phase of the work, for the correct solution of any problem depends on an accurate statement of the conditions of the problem and the expression of those conditions in correct mathematical language.

Transposition and clearing of fractions, two of the most fundamental processes in the solution of the equation, must be approached through simple problems where a need for these processes will be apparent. The terminology can be gradually

introduced, but the fundamental principles that like changes must be made in both members of the equation to keep the balance must he adhered to. These processes are made very graphic by means of the balance or scale, and axioms and principles derived by the pupils themselves. Generalizations thus derived are of more permanent value than "cut and dried" facts handed out to pupils.

If beginners in the study of algebra could feel that they were investigators in a new field of activty, and felt the realization of accomplished aims, we would have more cooperation between teacher and pupil in working out desired ends and necessarily more satisfactory results.

Baton Rouge banks are charging 10 cents exchange on checks drawn on out-of-town banks. One-dollar subscription check nets only 90 cents. By sending several subscriptions under one check a great saving is effected.

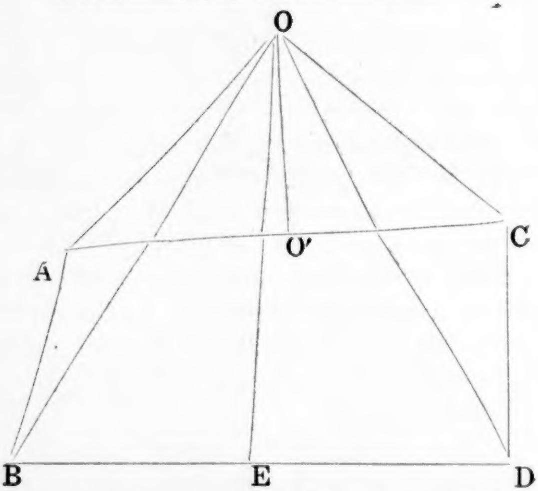
WHERE IS THE FALLACY?

By PROFESSOR I. MAIZLISH
Centenary College

The absurdities into which one is led when divison by zero is conceded to be an admssible operation is sometimes illustrated by the familiar example of "proving" that 1 is equal to 2. The absurdity in examples of this sort arises because an "order"—not to divide by zero—has been disobeyed.

At one of the meetings of the Mathematics Club of Centenary College one of the students cited this problem as one of the "puzzles" to which part of the meeting was devoted. I immediately recalled that while a sophomore in high school I was given a "puzzle" of a different sort from that usually found in text-books. It is, of course, not original with me, and I do not know where it is to be found. In the belief that it will interest teachers as well as students of geometry I have taken the liberty of submiting it to the News-Letter.

Problem: To Prove that a **right angle** is equal to **an acute angle.**



Construction: Draw any line BD. From pt. B lay off any distance AB, and then draw DC equal to AB and perpendicular to BD. Draw AC, and erect perpendicular bisectors OO′ to AC, and OE to BD. Draw OA, OC, OB, and OD.

To Prove: That <ABD is equal to right <CDB.

Proof: AB=CD (By construction)

OA=OC (Every pt. in the perpendicular bisector of a line is equi-distant from the extremities of that line).

OB=OD

and <OBD=<ODB (In an isosceles triangle the angles opposite the equal sides are equal).

Now in triangles BAO and DCO, we have

OA=OC,

OB=OD,

and AB=CD.

Therefore, these triangles are equal, since three sides of one triangle are respectively equal to three sides of the other.

Hence, <ABO=<CDO.

So that,

<ABO+<OBD=<CDO+<ODB If equals be added to equals, the results are equal).

But

<ABO+<OBD=<ABD, and <CDO+<ODB=rt.<CDB.

Therefore,

Acute<ABD=Right <CDB. *Q. E. D.*

THE MATHEMATICIAN AND THE CARPENTER

By IRBY C. NICHOLS
Louisiana State University

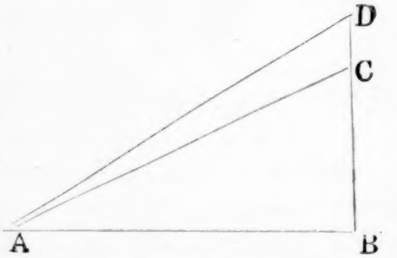
(Written expressly for students of High School geometry)

Referring again to vacations and their interesting and profitable moments, it may be in order to tell of a precise teacher of college mathematics who spent his summer with a group of plain house builders: carpenters, bricklayers, plumbers and others.

Theoretical mathematics in association with applied mathematics! Notice, please, the words "in association with." We do not say, "opposed to", "versus", or "in contrast to"; we do not have in mind the opposition of **theoretical** mathematics to **practical** mathematics, but we imply an alliance, a **co-operation,** a **harmonious** and **benevolent contact** of the one with the other.

Professor Bookie Brown contracted with Mr. Frank Carpenter to build for him a five-room residence. These two people were of fairly normal personalities and, therefore, were able to work together harmoniously. Wherein they were of mutual help to each other can best be told by way of some illustrations:

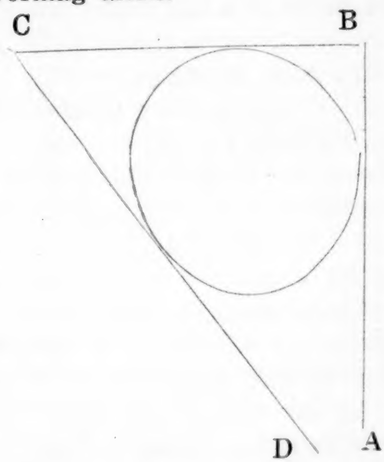
(1) In planning the house the matter of the pitch of the roof came up for discussion. Mr. Carpenter, in his frank way, advocated a pitch of $5\frac{1}{2}$ inches in 12 inches, "this pitch being **sufficient,** and **economical,** and, moreover, **common practice.**" This last argument "common practice should have closed the argument, particularly since college professors are not generally thought to have any practical ideas of their own. But Mr. Carpenter, having had little or no text book training, was not so thoroughly familiar with the exact laws governing the relative areas of triangles, and of how to get the number of degrees in angles from known values of their tangents. Professor Brown, trained in text-book mathematics, could see right triangles, tangents, degrees of angles, and lengths of sides all in one mental flash. He reached for his book of tables and, in a few minutes, knew that the base acute angle A of a right tri-



angle ABC whose legs are $5\frac{1}{2}$ and 12 has a tangent of .4583 and, that angle $A=24°37'$. He constructed such a triangle to scale, held it up before him and looked at it. Likewise he drew triangles representing pitches of 6 in 12 and of 7 in 12 respectively. The corresponding angles he found to be $26°34'$ and $30°15'$ respectively. He decided that $30°15'$ looked best to him —it was neither too steep nor too flat. But Mr. Carpenter raised the question of increased cost, cost being a factor of very great importance in house-building. However, he seemed to have the impression that the lengths of rafters increased in proportion to the increase in the elevation of the roof, and, that therefore costs increased at the same rate. Professor Brown knew exactly the law involved and that it had to do with the square roots of the sums of the squares of the legs of the right triangles respectively. Figuring a little, he found that, if the pitch be $5\frac{1}{2}$ in 12, the length of the rafters would be 13.20 in 12; and if 7 in 12 then the rafter would be 13.89 in 12. The increase in the height of the roof would be 27.2 per cent, but the increase in the length of the rafters would be 5.2 per cent. Obviously the increase in the cost should not be a great deal. Making further calculations and reducing things to dollars and cents, Professor Brown doing the figuring and Mr. Carpenter furnishing the structural needs and market values of materials, they found that the total increase in the cost of making the proposed change in the roof would be only about $30.00. Then the question was: Should an extra $30.00 be spent just to satisfy Profesor Brown's feeling that a pitch of 7 in 12 looks better than $5\frac{1}{2}$ in 12? Mr. Carpenter argued against it—"money thrown away"—"$5\frac{1}{2}$ in 12 is a good pitch and all right; every body else uses it.'

However, another thought flashed through Professor Brown's mind. A man of his habits and needs should have lots of storage space. Would he not secure a good deal more space in his attic? His familiarity with mathematics prompted him to catch quickly and accurately the corresponding increase in volume of the space involved. "Solids vary as the products of their three

dimensions," he said. Moreover the clearance under the rafters would be much greater—about 9 1/3 feet as compared with 7 1/3 feet—and, by framing his roof in the shape of a cross, he would have space for three pretty nice little rooms. These could be finished off now, or left until money was easier. These three potential rooms for an extra $30.00! The facts were clear and convincing; Mr. Carpenter became an advocate of the 7 in 12 pitch. Mathematical theory and practice were working together, and doing better than when working alone.



(2) A second nice problem arose in planning space for a cylindrical water tank to be in a corner behind the kitchen door. The corner was produced by a closet wall AB, 24 inches long, meeting at right angles the outside wall BC of the kitchen. The kitchen door CD was to be 32 inches wide and to be hinged at C. The diameter of the tank was 15 inches. The tank was to be located to the right of the door, and it was desired to leave as much space as possible to the left of the door. Only the line BC could be changed. How long should BC be to produce the most satisfactory results? An examination of the facts revealed that the free edge of the door D could not be made to come entirely back against the closet wall AB, since the width of the door and the diameter of the tank were fixed values. A compromise would have to be effected. The professor and the carpenter, working together, arrived at the conclusion that BC should be about 23 inches. This would result in an opening of about 5 or 6 inches between the free edge D of the door and the outer corner A of the closet wall AB when the door stood opened to its maximum, that is tangent to the tank. As construction proceeded on the house, many other problems came up wherein the man of text books and the man of the saw and the hammer, by working together, produced results more satisfactory than either alone could have produced.

(3) An elliptic transom over the front door proved to be an interesting problem. As ordinarily constructed by carpenters, many details of such fronts are only approximate. Professor Brown very consistently inisisted upon mathematical accuracy. He secured it and, if the favorable comments of subsequent visitors be taken at a fair value, the effect was sufficiently pleasing to make worth while the extra pains and efforts required to secure text book accuracy.

(4) A disappearing stairway, made for ceilings not exceeding 9 feet, needed reconstruction for a ceiling 9 feet 7 inches, or else a stairway costing \$20.00 more would have to be substituted. Mathematics, theoretical and practical, saved the \$20.00.

(5) An order had to be placed with a factory for an iron railing for a concrete front porch, set with six columns of circular cross sections, the bottoms to be 8-inch circles, the top to be 6 inches; the elevation of the railing above the floor to be 4 inches and the total height of the railing to be 28 inches. Knowing the characteristics of iron, the question arose as to whether an order should be placed early for the railing, the order to be based on calculations from theoretical measurements; or should the house be finished, the columns be set, and then measurements be made from actual spaces, from which data the order was then to be made. The latter plan would insure a fit, but it would cause a delay of several weeks in finishing the house. Theoretical mathematics furnished the means of placing the order early and of forestalling a delay.

So it was that a certain college professor spent his vacation; and so it was that, without the aid of a professional architect, a cottage of more than passing attraction and convenience was constructed. So it also happened that certain carpenters and working men, far away from college halls, came to know that a college professor too has some value aside from his class room; and the college professor came back to his teaching knowing better, and therefore loving more, the members of an allied profession and bringing to his students a wealth of new problems and interesting illustrations of the practical uses of his class-room teachings.

Ten years experience in teaching high school mathematics should yield at least four good contributions to the News Letter.